TUF

math for all ages

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TUF IS AVALON HILL'S TRADEMARK NAME FOR ITS GAME OF MATHEMATICS

TUF – a brief introduction

TUF is a fast, competitive game based on number sentences or equations. It is meant to be played by two, three, or four players in a group. TUF is a great solitaire game, too. Concentration and logical thought are essential; excitement is generated by competition between opponents and time.

Everyone hates to wait! In most games, the play rotates from one competitor to the next and the play is too slow to ensure the attention of the players, particularly younger players. The game of TUF differs in that all of the players work individually and simultaneously to achieve a goal. Each player is striving against his opponents and against time. The game requires rapt attention and continuous effort on the part of each player.

TUF is a progressive series of games suitable for various ages and skills: seven to adult. The first games merely deal with the operations of addition and subtraction in simple two, three, or four number equations. The next games increase the types and complexity of the equations by introducing multiplication, division, parentheses, fractions, and decimals. Later games introduce number systems to bases other than decimal, exponents, including powers, fractional and negative roots.

Game equipment:

20 – Red Operator Cubes marked + – X ÷ (X) –

12 – Blue Number Cubes marked 0 1 2 3 4 5

12 – Green Number Cubes marked 6 7 8 9 10 0

8 – Yellow Fraction Cubes marked $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{8}$ $\frac{1}{10}$

4 – White Equals Cubes marked = = = = = =

4 – Orange Constant Cubes % π log j tan χ

4 – Blank Cubes

3 – Timers: Red 3 min., Yellow 2 min., Blue 1 min.

1 – Set Rules

1 – Game Container

RULES FOR TUF

There are a number of TUF games that can be played, each based on particular mathematical operations. We suggest you start with the Basic Game, become familiar with the play, then move on to the games that interest you. If you are not familiar with the mathematics of a particular game, we are sure you will find the examples accompanying each game sufficient for playing purposes.

GAME 1 – BASIC GAME

Two to four people can play Game 1.

Each player takes:

3 Blue number cubes	4 Red operator cubes
3 Green number cubes	1 White equals cube

On a signal from one player, each player rolls his cubes out on the table from his hands. The upper face is used – the cube cannot be turned after the throw.

The first person to form an equation, using as many cubes as he can, calls out, "TUF!" For typical equations, see examples on the following pages.

After declaring his equation, TUF starts a timer and removes his hands from the table. The others are allowed this time interval to try to form a longer equation than TUF's. TUF has this time to mentally try to think of a longer equation than the one he has. If one of the other players can form a longer equation, that is use more cubes than TUF's in the allotted time after TUF declares, he calls out, "TUFFER," and sets the second timer. TUF then has an opportunity with the other players to try to exceed the length of TUFFER's equation during the timer interval. He cannot change the face of the cubes, but he may re-arrange his cubes in any order he wishes. If anyone succeeds in beating TUFFER, he calls, "TUFFEST," and sets the next timer to allow the other players to review their equations and attempt to improve them. Again, the players cannot change the face of their cubes, but may re-arrange their cubes in any order they wish to form a valid equation. The game may continue ("MORE TUFFEST," "MOST TUFFEST") until either a player uses all his cubes or the allotted time interval runs out. The Blue timer is used again – the fractional remaining time is allowed to run before resetting.

The final declarer (with a correct equation) counts two points for each cube used including the equals cube. The equations formed by the other players are counted by allowing one point for each cube used. If a player is found to have an incorrect equation he counts zero for that round.

At the end of the time interval, the scores are calculated for that round and noted on a score sheet. The highest total score after five games is the winner. Totaling score over a number of games is important. Children quickly learn the importance of making a short equation for scoring purposes even though they haven't a winner.

The suggested use of the timers is: Red (3 min.) for TUF, Yellow (2 min.) for TUFFER, Blue (1 min.) for TUFFEST. For faster games, Yellow may be used for TUF, and Blue for TUFFER and TUFFEST. Or Blue can be used for TUF, TUFFER, TUFFEST. The fractional time left over is allowed to run and the timer is then reset by the declarer to give the full minute before "time" is called. Timers can also be combined to give longer intervals.

To avoid argument, as soon as TUF declares, he must remove and keep his hands off the table. He cannot change his equation in any way, even if he sees an error or a longer equation. The other players must take their hands off the table at the end of the time interval.

Equation Examples for Game 1

$\boxed{4}\ \boxed{=}\ \boxed{4}$

$\boxed{3}\ \boxed{-}\ \boxed{2}\ \boxed{+}\ \boxed{6}\ \boxed{=}\ \boxed{7}$

$\boxed{2}\ \boxed{-}\ \boxed{6}\ \boxed{-}\ \boxed{5}\ \boxed{=}\ \boxed{-}\boxed{9}$

$\boxed{4}\ \boxed{\times}\ \boxed{3}\ \boxed{=}\ \boxed{6}\ \boxed{+}\ \boxed{6}$

$\boxed{10}\ \boxed{\div}\ \boxed{-}\boxed{5}\ \boxed{\times}\ \boxed{3}\ \boxed{=}\ \boxed{-}\boxed{6}$

$\boxed{-}\boxed{2}\ \boxed{=}\ \boxed{-}\boxed{2}$

$\boxed{3}\ \boxed{\times}\ \boxed{-}\boxed{2}\ \boxed{=}\ \boxed{-}\boxed{6}$ or $\boxed{-}\boxed{3}\ \boxed{\times}\ \boxed{2}\ \boxed{=}\ \boxed{-}\boxed{6}$

$\boxed{8}\ \boxed{\times}\ \boxed{0}\ \boxed{=}\ \boxed{0}\ \boxed{\times}\ \boxed{-}\boxed{3}$

$\boxed{10}\ \boxed{\div}\ \boxed{5}\ \boxed{\times}\ \boxed{3}\ \boxed{=}\ \boxed{6}$

$\boxed{10}\ \boxed{-}\ \boxed{4}\ \boxed{\div}\ \boxed{2}\ \boxed{=}\ \boxed{8}$

To give more variables and add interest, one number is allowed to be formed from two cubes:

$$\boxed{4}\ \boxed{\times}\ \boxed{7}\ \boxed{=}\ \boxed{2}\boxed{8}$$

$$\boxed{1}\boxed{5}\ \boxed{-}\ \boxed{8}\ \boxed{=}\ \boxed{7}$$

$$\boxed{3}\ \boxed{\times}\ \boxed{10}\ \boxed{=}\ \boxed{3}\boxed{0}$$

$$\boxed{10}\ \boxed{\times}\ \boxed{10}\ \boxed{+}\ \boxed{4}\ \boxed{+}\ \boxed{1}\ \boxed{=}\ \boxed{10}\boxed{5}$$

Order of Arithmetic Operations

First do all multiplication and division in order from left to right. Then the addition and subtraction is done in any order.

$$\boxed{8}\ \boxed{-}\ \boxed{6}\ \boxed{\times}\ \boxed{2}\ \boxed{=}\ {}^{\boxed{-}}\boxed{4}$$

$$\boxed{8}\ \boxed{-}\ {}^{\boxed{-}}\boxed{6}\ \boxed{\times}\ \boxed{2}\ \boxed{=}\ \boxed{2}\ \boxed{\times}\ \boxed{10}$$

$$\boxed{8}\ \boxed{\div}\ {}^{\boxed{-}}\boxed{2}\ \boxed{=}\ {}^{\boxed{-}}\boxed{4}$$

$$\boxed{10}\ \boxed{-}\ \boxed{10}\ \boxed{\div}\ \boxed{2}\ \boxed{=}\ \boxed{5}$$

$${}^{\boxed{-}}\boxed{8}\ \boxed{\div}\ {}^{\boxed{-}}\boxed{2}\ \boxed{=}\ \boxed{4}$$

$${}^{\boxed{-}}\boxed{8}\ \boxed{\div}\ \boxed{2}\ \boxed{=}\ {}^{\boxed{-}}\boxed{4}$$

Note that for game 1, there are two multiplication signs: x and (x) on the red cubes. There are two zeros: 0 on the blue blocks and θ on the green blocks.

The positive or plus sign can only be used where normal mathematical usage requires it. Redundant or unnecessary use of plus + is not allowed.

Examples

$\boxed{-}\boxed{8}\boxed{\div}\boxed{-}\boxed{4}\boxed{=}\boxed{2}$, *not* $\boxed{+}\boxed{2}$

$\boxed{9}\boxed{-}\boxed{5}\boxed{=}\boxed{4}$;

$\boxed{+}\boxed{9}\boxed{-}\boxed{5}\boxed{=}\boxed{4}$ is unacceptable as $\boxed{+}$ is redundant

The *negative* sign is indicated by raising the $\boxed{-}$ cube slightly above the others. When *subtract* or *minus* is to be shown the $\boxed{-}$ cube is placed in line with the number cubes:

$\boxed{-}\boxed{5}\boxed{+}\boxed{9}\boxed{=}\boxed{4}$ or $\boxed{9}\boxed{+}\boxed{-}\boxed{5}\boxed{=}\boxed{4}$

and $\boxed{-}\boxed{2}\boxed{-}\boxed{-}\boxed{5}\boxed{=}\boxed{3}$

Because *equating to zero* can become an over-used gimmick, it can be disallowed if agreed on before the game starts, or discouraged with a two point penalty (two cube penalty) deduction. If TUF formed an equation with 11 cubes by equating to zero, his equation would only have a 9 cube value. TUFFER could then declare with only ten cubes.

GAME 1A – TUF FOR VERY YOUNG PLAYERS

For pre-school and grades 1 and 2 children, TUF can be played using addition and subtraction operations only. This primary game can be played in several ways:

(a) Using the red operator cubes as constants, they may be set at either 4 +, or 2 + and 2 −, or 1 + and 3 −, or 4 −.

(b) Rolling the red operator cubes with the number cubes as in the Basic Game but before the play begins, changing the multiplications signs x and (x) to addition +, and the division signs ÷ to subtraction.

GAME 1B – CUTTHROAT TUF

For hardened veterans of the TUF arena, a more aggressive and competitive version can be played by making a small change in the system of scoring. When the declarer of the moment TUF, TUFFER, or TUFFEST is bested by another player, his score reverts to zero instead of to a single count. He can score again only by making a new equation with a count equal to TUFFER (or TUFFER to TUFFEST), during the allotted time interval. He then counts a single score. Or better yet, he can beat the new declarer and call TUFFEST (or TUFFEST AGAIN) and return to his double score position.

If TUF, TUFFER, or TUFFEST have an *incorrect equation* and the error is recognized by another player he can call out, "TUF ERROR", and if all agree that TUF is in error the discoverer scores the equivalent of all the blocks used by the incorrect declarer. On the other hand, if the error discoverer is found to be the one in error, he deducts a like amount.

This method of scoring injects a new element of risk (and caution!) which adds spice to the game for experienced players. It also tends to temper the impetuous, quick thinking player and encourages longer, more complicated equations.

GAME 1C – SOLITAIRE

Several versions of solitaire can be devised:

(a) For a given roll of cubes, the goal is to make the longest equation possible in a given time, such as five minutes.

(b) Larger numbers of blocks could be used: 4 blues, 4 greens, 6 reds, and 2 yellows with a longer interval of time allowed such as 15 minutes. The special incentive points (see later games 2 to 6) could be used to encourage more difficult equations.

(c) For a type of double solitaire with two players, one set of cubes is rolled out. A second set is required for the second player and these cubes are set to duplicate the original roll. Then on the word 'go', each player tries to create the longest (or highest point score if incentive points are used – see later games 2 to 6) equation in a given time: 2, 4, or 6 minutes. The advantage of using the incentive, or bonus points, is that a player is not sure if he has a winning score even when he incorporates all the cubes into equations.

GAME 2 – FRACTIONS

This is the same as Game 1 with the addition of one yellow fraction block for each player and also one red operator cube, giving five instead of four. The cubes used are:

- 3 Blue number cubes
- 1 Yellow fraction cube
- 1 White equals cube
- 3 Green number cubes
- 5 Red operator cubes

To encourage the use of the fraction cube, it should be assigned a two cube value (to be doubled to four for the final declarer, TUF, TUFFER, or TUFFEST).

Examples:

$\frac{1}{8} \times 8 = 1$

$7 - 4 \times \frac{1}{3} \times 6 = {}^{-}1$

$10 \div \frac{1}{10} = 10 \times 10$

${}^{-}9\frac{1}{5} - 4 \div 5 = {}^{-}10$

$7 - \frac{1}{10} \times 20 = 5$

$4 \div \frac{1}{3} + 2 - 4 = 10$

$9\frac{1}{5} + 4 \div 5 = 10$

As shown in the last examples, a fractional number like $9\tfrac{1}{5}$ can be formed by combining one or two integer cubes with a fraction cube. In addition, one whole number can also be formed from two integer cubes, such as $\boxed{2}\boxed{4}$ and $\boxed{10}\boxed{0}$. An example of two integer cubes and a fraction is $\boxed{10}\boxed{0}\boxed{\tfrac{1}{2}}$,

Note that *dividing by a fraction* is the same as multiplying by the fraction upside down (the upside down fraction is called the reciprocal).

$5 \div \tfrac{1}{4}$ is the same as $5 \times \tfrac{4}{1} = 20$

$6 \div \tfrac{2}{3}$ is the same as $6 \times \tfrac{3}{2} = 9$

GAME 2A

Each player uses two yellow fraction cubes. This greatly increases the possible fractional combinations and game complexity. If preferred, the integer (number) cubes can be reduced to two blues and two greens.

$$\boxed{\tfrac{1}{2}}\ \boxed{\times}\ \boxed{\tfrac{1}{3}}\ \boxed{\times}\ \boxed{2}\ \boxed{=}\ \boxed{3}\ \boxed{\div}\ \boxed{9}$$

$$\boxed{\tfrac{1}{2}}\ \boxed{-}\ \boxed{\tfrac{1}{8}}\ \boxed{=}\ \boxed{3}\ \boxed{\div}\ \boxed{8}$$

$$\boxed{-}\boxed{\tfrac{1}{5}}\ \boxed{\div}\ \boxed{\tfrac{1}{8}}\ \boxed{\times}\ \boxed{10}\ \boxed{=}\ \boxed{-}\boxed{2}\ \boxed{\times}\ \boxed{8}$$

$$\boxed{\tfrac{1}{3}}\ \boxed{+}\ \boxed{\tfrac{1}{8}}\ \boxed{=}\ \boxed{2}\boxed{2}\ \boxed{\div}\ \boxed{8}\ \boxed{\div}\ \boxed{6}$$

GAME 3 – DECIMAL POINT

Game 3 is the same as Game 2 with only one change: the zero with the dot 0̇ on the green cubes *must* be used as a decimal point only.

When using the decimal point there is no restriction on the number of numerals incorporated into a decimal number as long as normal mathematical convention is followed and the equation is correct.

Examples: *(Normal Mathematical Notation on top – Equivalent TUF underneath.*

$\frac{1}{2} = .5$

$\boxed{\tfrac{1}{2}}\ \boxed{=}\ \boxed{\theta}\boxed{5}$

$\frac{1}{2} = 0.5$ or $= 0.50$

$\boxed{\tfrac{1}{2}}\ \boxed{=}\ \boxed{0}\boxed{\theta}\boxed{5}$ or $\boxed{=}\ \boxed{0}\boxed{\theta}\boxed{5}\boxed{0}$

$3 \div 10 = 0.3$

$\boxed{3}\ \boxed{\div}\ \boxed{10}\ \boxed{=}\ \boxed{0}\boxed{\theta}\boxed{3}$

$\frac{1}{10} \times 4 \times 2 = .8$

$\boxed{\tfrac{1}{10}}\ \boxed{x}\ \boxed{4}\ \boxed{x}\ \boxed{2}\ \boxed{=}\ \boxed{\theta}\boxed{8}$

$^{-}3 \div .1 = {}^{-}6 \div \frac{1}{5}$

$\boxed{-}\boxed{3}\ \boxed{\div}\ \boxed{\theta}\boxed{1}\ \boxed{=}\ \boxed{-}\boxed{6}\ \boxed{\div}\ \boxed{\tfrac{1}{5}}$

$5.5 + \frac{1}{2} = 6$

$\boxed{5}\boxed{\theta}\boxed{5}\ \boxed{+}\ \boxed{\tfrac{1}{2}}\ \boxed{=}\ \boxed{6}$

$^{-}9 \div 0.1 = {}^{-}90$

$\boxed{-}\boxed{9}\ \boxed{\div}\ \boxed{0}\boxed{\theta}\boxed{1}\ \boxed{=}\ \boxed{-}\boxed{9}\boxed{0}$

$9.125 = 9 + \frac{1}{8}$ or $= 9\frac{1}{8}$

$\boxed{9}\boxed{\theta}\boxed{1}\boxed{2}\boxed{5}\ \boxed{=}\ \boxed{9}\ \boxed{+}\ \boxed{\tfrac{1}{8}}$ or $\boxed{=}\ \boxed{9}\boxed{\tfrac{1}{8}}$

GAME 4 – PARENTHESES

The use of parentheses () is added to the previous games to allow grouping of operations and numbers. The red operator cubes have one face marked (x) and this can be () parentheses or multiplication, as desired. If brackets [] and braces {} are required, they can also be represented by (x).

In order to overcome the problem of providing an *opening and a closing parenthesis* as required in normal mathematical notation, we place both the parentheses, as shown on the cube face, before the quantity to be grouped, or if preferred, after the group. We assume the group is closed at the other end; to show this a *space is left* between the group and the following (or preceding) cubes.

Now any number or operator placed beside the parentheses multiplies everything in the group except where ÷ is the indicated operation. Where the operators + or − precede the parentheses, a 1 is assumed present and the group multiplied by +1 or ⁻1.

Note that the quantity within the parentheses is calculated first.

Examples:

$\frac{1}{5}\ (6 + 4) = 2$

[⅕] [(x)] [6] [+] [4] [=] [2]

$(6 + 4) \div \frac{1}{8} = 80$

[6] [+] [4] [(x)] [÷] [⅛] [=] [8] [0]

$(2 + .1)\ 5 = 10\frac{1}{2}$

[2] [+] [0] [1] [(x)] [5] [=] [1] [0] [½]

$(3 + 9 - .5) \div \frac{1}{8} = 92$

[3] [+] [9] [−] [0] [5] [(x)] [÷] [⅛] [=] [9] [2]

$\frac{1}{3}\ (9 \div 6) + 2 = 2.5$

[⅓] [(x)] [9] [÷] [6]   [+] [2] [=] [2] [0] [5]

$5\ [(9 - 4)\ 2 + 10] = 10 \div \frac{1}{10}$

[5] [(x)] [9] [−] [4] [(x)] [2] [+] [10] [=] [10] [÷] [⅒]

or [5] [(x)] [2] [(x)] [9] [−] [4]   [+] [10] [=] [10] [÷] [⅒]

GAME 5 – BASES OTHER THAN DECIMAL

Number systems to bases other than our decimal system (base 10) may be used to form equations. The base of the number system is shown by placing the cube indicating the base below the line and to the right of the number. However, where practice in bases is the main interest, an integer cube is not used to indicate base; the players merely *state the base* used in their equation. This greatly increases the combinations possible with a given set of cubes. All other numbers without the base indicated or stated, are decimal.

When the standard TUF game is being played incorporating Game 5, a one extra cube bonus can be allowed (two for final TUFFER) for any equation incorporating non-decimal numbers. We suggest that standard TUF play require a base indicator cube. The players can, however, decide beforehand to merely state the base of a non-decimal number.

As many cubes as required may be placed together to form a non-decimal number. The usual two cube decimal number is also allowed.

Examples:

Binary or Base 2	$\boxed{1}\boxed{1}\boxed{0}_{\boxed{2}}\ \boxed{=}\ \boxed{6}$
Base 3	$\boxed{2}\boxed{0}\boxed{1}_{\boxed{3}}\ \boxed{=}\ \boxed{10}\ \boxed{+}\ \boxed{9}$
Base 12	$\boxed{8}\boxed{8}_{\boxed{1}\boxed{2}}\ \boxed{=}\ \boxed{10}\boxed{4}$

For you who have forgotten or are not familiar with numbering systems other than decimal (base 10), the following is a very brief synopsis sufficient for playing TUF.

Whether a number is decimal or to another base, the place value indicates the quantity represented by each digit position. Read the digits from right to left and multiply each by the position or *place value*. The first place value is base to the zero power or B^0, second is B^1, third is B^2, ... etc. Thus the base power is one less than its position.

To illustrate this, the following are examples of decimal (base 10), binary (base 2), base 3 and base 5.

Decimal or Base 10: 201 decimal

$$\begin{array}{ccccc} 2 & & 0 & & 1 \\ \text{x} & & \text{x} & & \text{x} \\ 10^2 & & 10^1 & & 10^0 \\ \text{or} & & \text{or} & & \text{or} \\ \underline{100} & & \underline{10} & & \underline{1} \\ 200 & + & 0 & + & 1 = 201_{\text{decimal}} \end{array}$$

Binary or Base 2: 101_2

$$\begin{array}{ccccc} 1 & & 0 & & 1 \\ \text{x} & & \text{x} & & \text{x} \\ 2^2 & & 2^1 & & 2^0 \\ \text{or} & & \text{or} & & \text{or} \\ \underline{4} & & \underline{2} & & \underline{1} \\ 4 & + & 0 & + & 1 = 5_{\text{decimal}} \end{array}$$

Base 3: 201_3

$$\begin{array}{ccccc} 2 & & 0 & & 1 \\ \text{x} & & \text{x} & & \text{x} \\ 3^2 & & 3^1 & & 3^0 \\ \text{or} & & \text{or} & & \text{or} \\ \underline{9} & & \underline{3} & & \underline{1} \\ 18 & + & 0 & + & 1 = 19_{\text{decimal}} \end{array}$$

Base 5: In the above examples we changed from a known base not decimal, to a decimal base number. In this example we will take a decimal number and change it to the equivalent base 5.

Given 105 decimal, change to a base 5 number. First analyze base 5 numbers by finding each digit's decimal equivalent place value from right to left. Calculate all place values to the point where the value is higher than that of the decimal number which is to be changed.

Place 4	*Place 3*	*Place 2*	*Place 1*
5^3	5^2	5^1	5^0
or	or	or	or
125	25	5	1

Then start dividing the place values into the decimals and its remainders. Begin at the first place smaller than the decimal.

$105 \div 25$	$5 \div 5$	$0 \div 1$
$\underline{5}$		
4	1	$0 = 410_5$

Thus: $105 = 405_5$ or $\boxed{10}\boxed{5}\ \boxed{=}\ \boxed{4}\boxed{10}_{\boxed{5}}$

GAME 6 – EXPONENTS OR POWERS AND ROOTS

The use of exponents or powers and roots is now included. The game is similar to games 2, 3 and 4 and the same cubes are used.

The normal mathematical notation is used. The exponent cubes are merely placed above and to the right of the base number. Integer cubes are powers and fraction cubes are roots. Compound exponents can also be assembled from various cubes.

As an incentive to the use of exponents, all cubes used in an exponent may be given a two cube value (which becomes four for the successful declarer). This includes red operator cubes as well. Since the yellow fraction cubes are already worth two, they retain this value when used as an exponent (or four for the TUFFER). This extra value on exponents is valuable where players wish practice in the use of exponents. Otherwise players may wish to leave all exponents at a single count, except the yellow fraction cube which always has a two cube value.

Examples:

$\boxed{8}^{\boxed{2}}\ \boxed{=}\ \boxed{6}\boxed{4}$ $\qquad$ $\boxed{8}^{\boxed{1/3}}\ \boxed{=}\ \boxed{2}$

Fractional and decimal exponents can be assembled from several cubes.

$$\boxed{2}\boxed{7}^{\boxed{2}\boxed{\div}\boxed{3}} \quad \text{or} \quad \boxed{2}\boxed{7}^{\boxed{1/3}\boxed{\times}\boxed{2}}$$

$$= \sqrt[3]{27^2} \quad \text{or} \quad (\sqrt[3]{27})^2 = 9$$

$$\boxed{1}\boxed{6}^{\boxed{1}\boxed{\odot}\boxed{5}} \quad \text{or} \quad \boxed{1}\boxed{6}^{\boxed{3}\boxed{\div}\boxed{2}}$$

$$= (\sqrt[2]{16})^3 \quad \boxed{=}\boxed{6}\boxed{4}$$

A base to the 0 power = 1 ex: $8^0 = 1$ and $0.56^0 = 1$

A base to the power of 1 equals itself ex: $5^1 = 5$

A number with a negative exponent is the same as the reciprocal of that number with the same positive exponent.

Examples:

4^{-2} is the same as $\dfrac{1}{4^2}$ or $= 1/16$

$8^{-1/3}$ is the same as $\dfrac{1}{8^{1/3}}$ or $\dfrac{1}{\sqrt[3]{8}}$ or $= 1/2$

$8^{-2\div 3}$ is the same as $8^{-2/3}$ or $\dfrac{1}{8^{2/3}}$ or $\dfrac{1}{(\sqrt[3]{8})^2}$ or $= 1/4$

$8^{-1\div(x)^2+1}$ is the same as $8^{-1\div(2+1)}$ or $8^{-1/3} = 1/2$

With exponents the order of mathematical operations is: Exponents are solved first (unless the exponent acts upon a group enclosed in parentheses – here the parentheses quantity is solved first and becomes the base number on which the exponent acts), then group quantities enclosed in parentheses, followed by multiplication and division from left to right, and finally addition and subtraction in any order.

Examples:

$$8^{\frac{1}{3}} = 2\,(6-5)$$

$$\boxed{8}^{\boxed{\frac{1}{3}}}\ \boxed{=}\ \boxed{2}\ \boxed{(x)}\ \boxed{6}\ \boxed{-}\ \boxed{5}$$

$$64^{-\frac{1}{3}} = 2 \div 8$$

$$\boxed{6}\,\boxed{4}^{\boxed{-}\,\boxed{\frac{1}{3}}}\ \boxed{=}\ \boxed{2}\ \boxed{\div}\ \boxed{8}$$

$$(8 \times 8)^{\frac{1}{3}} = 4 \div 1$$

$$\boxed{8}\ \boxed{\times}\ \boxed{8}\ \boxed{(x)}^{\boxed{\frac{1}{3}}}\ \boxed{=}\ \boxed{4}\ \boxed{\div}\ \boxed{1}$$

$$^{-}(8^{-\frac{1}{3}}) = {}^{-}0.5$$

$$\boxed{-}\,\boxed{(x)}\,\boxed{8}^{\boxed{-}\,\boxed{\frac{1}{3}}}\ \boxed{=}\ {}^{\boxed{-}}\boxed{0}\,\boxed{0}\,\boxed{5}$$

$$4 \times 5^{3} = (10 \times 10) \div \tfrac{1}{5}$$

$$\boxed{4}\ \boxed{\times}\ \boxed{5}^{\boxed{3}}\ \boxed{=}\ \boxed{10}\ \boxed{\times}\ \boxed{10}\ \boxed{(x)}\ \boxed{\div}\ \boxed{\frac{1}{5}}$$

$$[(5 \times 5 + .6)\,10]^{\frac{1}{8}} = 2$$

$$\boxed{5}\ \boxed{\times}\ \boxed{5}\ \boxed{+}\ \boxed{0}\,\boxed{6}\ \boxed{(x)}\ \boxed{10}\ \boxed{(x)}^{\boxed{\frac{1}{8}}}\ \boxed{=}\ \boxed{2}$$

THE CONSTANT CUBE

The addition of the Orange 'Constant' cube to the basic set opens up new possibilities for games and practice. The Constant cube is marked with six symbols: % π log tan χ j. This cube can be *preset* to a particular symbol or function on which practice or familiarization is required. It would not be rolled with the other cubes. When the players are conversant with the meaning and use of all the symbols on the cube, it can then be rolled with the other cubes during play of the standard TUF game.

Where the required symbol is to be preset and not rolled, a useful variation on the game play is to play against time only – equation length or cube quantity does not count for extra points. The first person to achieve a valid equation incorporating the symbol on the preset constant cube calls TUF and sets the timer. The other players have the timer interval to obtain an equation. The winner scores one more than the number of players; for four players he counts 5, The second player calling a correct equation counts two less than the winner, or 3, the third counts 2 and the fourth 1. A player not finding an equation counts 0. Incorrect equations merely count zero – there is no penalty. Five games can be a set or, if the time is short, the play is for a given time interval with the points totalled at the end of that period. This method of play is preferred for quick games giving maximum practice in a limited amount of time on a particular operation or function.

Examples covering each symbol are given below:

Percentage

$20\% = 10 \div 50$

[2][0] [%] [=] [10] [÷] [5][0]

$\frac{1}{2}\% = .5 \div 100$

[½] [%] [=] [0][5] [÷] [10][0]

$80\% \times 50 = 4 \div \frac{1}{10}$

[8][0] [%] [x] [5][0] [=] [4] [÷] [1/10]

$87\frac{1}{2}\% = \frac{1}{8} \times 7$, or $= 1.0 - \frac{1}{8}$

[8][7][0][5] [%] [=] [1/8] [(x)] [5] [+] [2]

Ratio and Proportion games can be played at the same time that Percentage games are being played. A different approach to the use of TUF cubes is taken. The ratio equation form is used with the ratio or division lines '— = —' assumed (a card with the lines marked on can be used if the players consider that it helps clarify the ratio representation). The cubes are then arranged in any way possible above and below the imaginary right and left hand ratio lines to form a valid ratio; either simple or compound.

$$\frac{\boxed{4}}{\boxed{8}}\ \boxed{=}\ \frac{\boxed{5}\,\boxed{\div}\,\boxed{1/10}}{\boxed{10}\,\boxed{0}} \qquad \frac{\boxed{1/8}}{\boxed{8}}\ \boxed{=}\ \frac{\boxed{4}\,\boxed{\div}\,\boxed{4}}{\boxed{2}^{\boxed{6}}}$$

Logarithm or log can be defined simply: where $3^4 = 81$, then $\log_3 81 = 4$. This is read: the base 3 log of 81 equals 4. Log is the exponent; base is the same for both.

$2^5 = 32$ then $\log_2 32 = 5$

$2^{-5} = 1/32$ then $\log_2 1/32 = -5$

$\log_2 1 \div (8 \times 4) = -5$

$\boxed{\log}_{\boxed{2}}\ \boxed{1}\ \boxed{\div}\ \boxed{(x)}\ \boxed{8}\ \boxed{\times}\ \boxed{4}\ \boxed{=}\ \boxed{-}\boxed{5}$

$\log_{10} 10^3 = 9 \times 1/3$

$\boxed{\log}_{\boxed{10}}\ \boxed{10}^{\boxed{3}}\ \boxed{=}\ \boxed{9}\ \boxed{\times}\ \boxed{1/3}$

$\log_{10} .01 = -(1 + 1)$

$\boxed{\log}_{\boxed{10}}\ \boxed{0}\boxed{0}\boxed{1}\ \boxed{=}\ \boxed{-}\boxed{(x)}\ \boxed{1}\ \boxed{+}\ \boxed{1}$

j or i the $\sqrt{-1}$

The $\sqrt{-1}$ is represented by the symbol j (i is also used to represent $\sqrt{-1}$). It is a convenient term for a difficult number.

i (or j) is useful in several phases of mathematics such as trigonometry and complex numbers. Some simple examples showing how j may be used in TUF play follow.

$$j = \sqrt{{}^{-}1} \text{ or } = ({}^{-}1)^{1/2}$$

$$\boxed{j}\ \boxed{=}\ \boxed{-}\boxed{1}\ \boxed{(x)}^{\boxed{1/2}}$$

$$j^2 = {}^{-}1$$

$$\boxed{j}^{\boxed{2}}\ \boxed{=}\ \boxed{-}\boxed{1}$$

$$j^3 = j^2 \times j = {}^{-}1 \times j = {}^{-}j = {}^{-}({}^{-}1)^{1/2}$$

$$\boxed{j}^{\boxed{3}}\ \boxed{=}\ \boxed{-}\quad \boxed{-}\boxed{1}\ \boxed{(x)}^{\boxed{1/2}}$$

$$j^4 = 1$$

$$\boxed{j}^{\boxed{4}}\ \boxed{=}\ \boxed{1}$$

$$j^6 = j^4 \times j^2 = {}^{-}1$$

$$\boxed{j}^{\boxed{6}}\ \boxed{=}\ \boxed{-}\boxed{1}$$

$$j^{75} = (j^4)^{18} \times j^3 = j^3 = {}^{-}j = {}^{-}\sqrt{{}^{-}1}$$

$$\boxed{j}^{\boxed{7}\boxed{5}}\ \boxed{=}\ \boxed{-}\quad \boxed{-}\boxed{1}\ \boxed{(x)}^{\boxed{1/2}}$$

$$\sqrt{{}^{-}36} = j\ 6$$

$$\boxed{-}\boxed{3}\boxed{6}\ \boxed{(x)}^{\boxed{1/2}}\ \boxed{=}\ \boxed{j}\boxed{6}$$

$$({}^{-}10 - 10)^{1/2} = j\ 2 \times 5^{.5}$$

$$\boxed{-}\boxed{10}\ \boxed{-}\ \boxed{10}\ \boxed{(x)}^{\boxed{1/2}}\ \boxed{=}\ \boxed{j}\boxed{2}\ \boxed{\times}\ \boxed{5}^{\boxed{0}\boxed{5}}$$

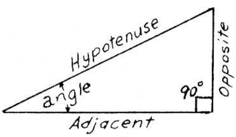
$$5 - (j2)^2 = 9$$

$$\boxed{5}\ \boxed{-}\ \boxed{j}\boxed{2}\ \boxed{(x)}^{\boxed{2}}\ \boxed{=}\ \boxed{9}$$

Note that j can also be used as a ‘joker’ to represent symbols which can be declared before the game starts: $<$, $>$, $\leq$, $\geq$, $\equiv$, $\neq$, etc. For Factorial turn j upside down. Thus $\boxed{4}\ \boxed{!} = 1 \times 2 \times 3 \times 4 = 24$.

Tangent or "tan"

The tangent symbol is included as a simple, brief introduction to trigonometric relations. The tangent is the most commonly used relationship in a right angled triangle. The tangent of the angle relates the size of the angle to the dimensions of the sides of the triangle. The tangent of the right angled triangle shown is defined as the *Opposite* side ÷ *Adjacent* side. It is written tan a = $\frac{0}{A}$ or tan a = 0 ÷ A.



The tangents of various angles can be looked up in standard tables, but listed below are the approximate tangents for a few angles which will be useful for playing TUF. Also, if we draw our triangle with the Adjacent = 1 (1 inch, 1 foot, 1 yard, etc.) then the tangent for a given angle will represent the relative length of the opposite side.

angle°	*tangent*	*angle°*	*tangent*
0	0	45	1
1	.017	60 (or $\frac{\pi}{3}$)	$\sqrt{3}$
10	.18	64	2.05
20	.36	76	4.01
30 (or $\frac{\pi}{6}$)	$\sqrt{3} \div 3$	87	19.
31	.60	88	29.
35	.70	89	57.
42	.90	90	∞

Integers following the tan symbol are always assumed to be degrees (unless π, representing radians, is used in later games – see "π").

$\tan\ (90 \div 2)^\circ = 2 \times \frac{1}{2}$

$\boxed{\tan}\ \boxed{(x)}\ \boxed{9}\boxed{0}\ \boxed{\div}\ \boxed{2}\ \boxed{=}\ \boxed{2}\ \boxed{x}\ \boxed{\frac{1}{2}}$

$\tan\ (8^2 - 4)^\circ = \sqrt{3}$

$\boxed{\tan}\ \boxed{(x)}\ \boxed{8}^{\boxed{2}}\ \boxed{-}\ \boxed{4}\ \boxed{=}\ \boxed{3}^{\boxed{\frac{1}{2}}}$

$\tan\ (7 \times 5)^\circ = .70$

$\boxed{\tan}\ \boxed{(x)}\ \boxed{7}\ \boxed{x}\ \boxed{5}\ \boxed{=}\ \boxed{0}\boxed{7}\boxed{0}$

$\tan\ (10 - 8 - 2)^\circ = 0$

$\boxed{\tan}\ \boxed{(x)}\ \boxed{10}\ \boxed{-}\ \boxed{8}\ \boxed{-}\ \boxed{2}\ \boxed{=}\ \boxed{6}\ \boxed{x}\ \boxed{0}$

$\tan^2 30^\circ = 1 \div 9^{\frac{1}{2}}$

$\boxed{\tan}^{\boxed{2}}\ \boxed{3}\boxed{0}\ \boxed{=}\ \boxed{1}\ \boxed{\div}\ \boxed{9}^{\boxed{\frac{1}{2}}}$

π can be used for the representation of angular dimension: π radians equals 180 degrees. It can also represent $\frac{22}{7}$ or 3.14 . . . . for geometry in the circular and spherical relationships.

π as Radians:

$\pi \div 3 = 60^\circ$

$\boxed{\pi}\ \boxed{\div}\ \boxed{3}\ \boxed{=}\ \boxed{6}\boxed{0}$

$.75\,\pi = (5^3 + 10)^\circ$

$\boxed{0}\boxed{7}\boxed{5}\ \boxed{\pi}\ \boxed{=}\ \boxed{5}^{\boxed{3}}\ \boxed{+}\ \boxed{10}$

or $\boxed{0}\boxed{7}\boxed{5}\ \boxed{x}\ \boxed{\pi}\ \boxed{=}\ \boxed{5}^{\boxed{3}}\ \boxed{+}\ \boxed{10}$

Examples (cont.)

$(10 \div 5)\ \pi = 360°$

$\boxed{10}\ \boxed{\div}\ \boxed{5}\ \boxed{(x)}\ \boxed{\pi}\ \boxed{=}\ \boxed{3}\boxed{6}\boxed{0}$

$\pi = \frac{22}{7}$ or 3.14 .... :

$4\ \pi = 88 \div 7$

$\boxed{4}\ \boxed{\pi}\ \boxed{=}\ \boxed{8}\boxed{8}\ \boxed{\div}\ \boxed{7}$

χ represents an unknown. The value of χ can be declared before the game begins and then is utilized, if possible, by the players.

It can also be allotted a bonus value such as two or three cubes. Several suggestions follow:

A) Fractions other than those on the yellow fraction cube:
$\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{9}$ $\frac{1}{12}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$ $\frac{1}{100}$ $\frac{1}{1000}$ $1/\sqrt{2}$ $1/\sqrt{3}$

B) Integers and other numbers of special significance: 12, 100, 1000, 10^6, $\sqrt{2}$, $\sqrt{3}$, e, etc.

C) Physical Constants:

$\frac{9}{5}$ (fahrenheit to centigrade degrees)

25.4 (mm. per inch)

28.41 (cc. per fluid oz.)

28.349 (gms. per oz. avoirdupois)

3412 (BTU. per kwhr.)

32.17 (acceleration of gravity in ft./sec.^{2})

0.621 (miles per kilometer)

2.205 (lbs. per kilogram)

The "SUPERLATIVE" Mathematics Game

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